

The mathematical techniques presented in this paper, although fairly abstract in concept, are computationally simple and do not take require a significant amount of time to solve for programs such as Mathcad. These programs can perform multi-dimensional matrix integration with complex exponentials in seconds with a click of a button. Going back and forth between time and frequency domains proves also to be simple and fast. In summary, the main task here is to write the equations that govern the circuit as accurately as possible. We leave it to the math software, along with its' powerful symbolic processor, to do the rest.

Confirmation of the integrity of our predictions is had by verifying that the predicted time domain waveforms converge to the actual bench test results with an ordinary oscilloscope (though some oscilloscopes have FFT capability making this task even easier). Once our results are verified to an acceptable level of tolerance, a good degree of certainty is also obtained with regard to its' frequency domain counterpart and thus its relevance to the IEC61000-3-2 standard.

The procedure we will use in this paper to determine the input harmonic levels are outlined as follows:

- Characterize the feed-forward signal relative to the line voltage in the time domain-Vff(t).
- Characterize the feedback signal relative to the line voltage, in the time domain-VEA(t).
- Characterize the multiplier block output and resulting input line current, Ipredict(t).
- Inject a realistic amount of zero-crossing error
- Calculate power factor, THD and harmonic levels and compare against IEC61000-3-2.
- Verify the integrity of the calculations by comparing all relevant time domain waveforms and measurements to actual bench test results.

II. FOURIER ANALYSIS OF THE FEED-FORWARD LOOP:

The purpose of this loop, although generally slow, is to as quick as possible adjust the PFC IC multiplier level relative to changes of the input line without having to wait for the even slower feedback loop to respond. This circuit's main responsibility is to filter the rectified line's dc component without overly compromising speed. A typical Feed-forward circuit utilizes a two pole network and is shown below in figure 1. In this section our objective is to plot the time domain waveform VFF(t) as shown in the figure. The input of this network is Vrect(t) -which is essentially the input rectified line.

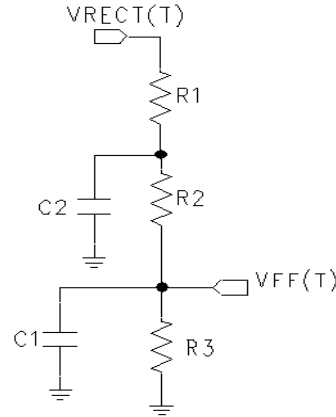


Figure 2-Feed-forward Topology

We begin by writing and plotting the input equation for this network (for Vin=230V & f=60Hz; See figure3 below).

$$\text{input}(t) := \left| \text{Vin} \cdot \sqrt{2} \cdot \sin\left(2 \cdot \pi \cdot \frac{t}{T_s}\right) \right| \tag{1}$$

For the DC component, or out₀, we integrate equation (1) over a full line cycle. Then using the exponential form of the Fourier Series integral² we calculate the input source coefficient matrix in equation (2) below: We then evaluate this matrix for harmonics (k=1 through 40th of the line frequency 60Hz) .

$$i_{n,k} := \frac{1}{T_s} \int_{0 \cdot \text{sec}}^{T_s} \left| \text{Vin} \cdot \sqrt{2} \cdot \sin\left(2 \cdot \pi \cdot \frac{t}{T_s}\right) \right| \cdot e^{-j \cdot k \cdot 2 \cdot \frac{\pi}{T_s} \cdot t} dt \tag{2}$$

We now have a complex matrix identifying the input source signal. We then algebraically derive the transfer function of the network shown in figure 1 and evaluate this network for K=1 through the 40th harmonic for Ts=1/60Hz and:

$$s_k := j \cdot 2 \cdot \frac{\pi}{T_s} \cdot k \tag{3}$$

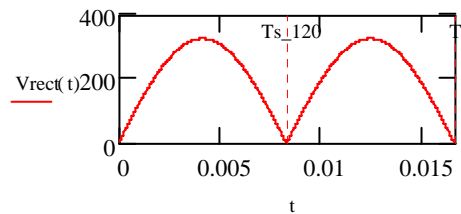


Figure 3- Input Line voltage -rectified.

$$T_{vk} := \frac{\left(\frac{\frac{R3}{s_k \cdot C1}}{\frac{1}{s_k \cdot C1} + R3} + R2 \right) \cdot \frac{1}{s_k \cdot C2}}{\left(\frac{\frac{R3}{s_k \cdot C1}}{\frac{1}{s_k \cdot C1} + R3} + R2 \right) + \frac{1}{s_k \cdot C2}} \cdot \frac{\left(\frac{R3}{s_k \cdot C1} \right)}{\left(\frac{R3}{s_k \cdot C1} + R2 \right)} + R1 \quad (4)$$

To get our output response, we now take the product of the complex source matrix with our feed-forward filter matrix:

$$out_k := in_k \cdot T_k \quad (5)$$

We then take the resulting output matrix and plug it into the time domain expansion below to get Vff(t) waveform.

$$V_{FF}(t) := out_0 + \sum_k \left(2 \cdot \text{Re}(out_k) \cdot \cos\left(2 \cdot k \cdot \frac{\pi}{T_s} \cdot t\right) \right) - \sum_k \left(2 \cdot \text{Im}(out_k) \cdot \sin\left(2 \cdot k \cdot \frac{\pi}{T_s} \cdot t\right) \right) \quad (6)$$

For R1=282K; R2=15K;R3=7.5K;C1=1uF; and C2=1uF; we get an output response as follows:

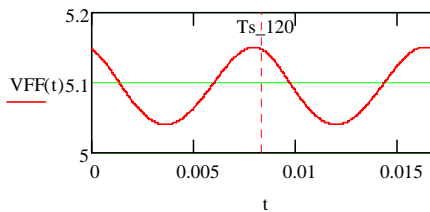


Figure 4-Feed-forward signal

The above waveform can be verified against actual results in order to confirm accuracy. The phase response relative to the rectified line is important as well and can be easily verified. This waveform eventually gets squared and

inverted before becoming an operand in the multiplier block.

III. FOURIER ANALYSIS OF THE FEEDBACK LOOP:

We now plot and analyze the PFC bus line frequency ripple and in the time domain and again use Fourier math to determine the feedback error amplifier output response. A typical feedback amplifier, and the one used for this paper, is shown in Figure 4 below:

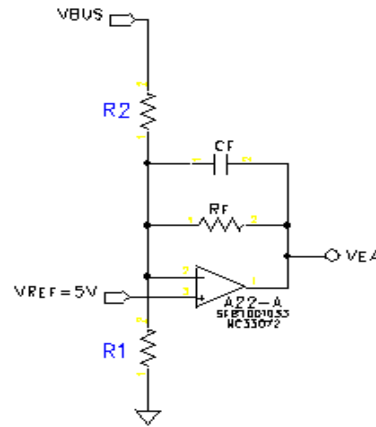


Figure 5- Feedback Error Amplifier

The line frequency ripple current through the output PFC capacitor (Co), IC_120(t), can be expressed as shown in equation (8) below. It is important to maintain the phase relationship between the input current and the output capacitor voltage since this current generates the bus ripple voltage sensed by the output error amplifier. Note that as was the case in the previous section, this phase information is vital in order to get an accurate representation of overall distortion.

$$IL_{60Hz_pk} := \frac{Pin_{pfc} \sqrt{2}}{V_{in}} \quad (7)$$

$$IL(t) := \left| IL_{60Hz_pk} \cdot \sin\left(2 \cdot \pi \cdot \frac{t}{T_s}\right) \right| \quad (8)$$

$$IC_{120}(t) := \frac{IL(t) \cdot V_{in} \cdot \left| \sin\left(2 \cdot \pi \cdot \frac{t}{T_s}\right) \right| \sqrt{2}}{V_o} \quad (9)$$

Now we plot the output PFC capacitor IC_120(t) in figure 7 below: This current is predominantly second harmonic and for the purposes of this analysis we will ignore any higher order harmonics here since they will be well filtered by the generally large output capacitance. (Here Pin=1205W;

$V_{in}=230V$ RMS; $V_{out}=400V$, $C_{actual}=1344\mu F$). Note that the average value of this current equals Pin_{pfc}/V_o as expected. This DC component is the current delivered to the load. The AC component results in a small ripple across the output capacitor due to the fact that it is usually very large in value.

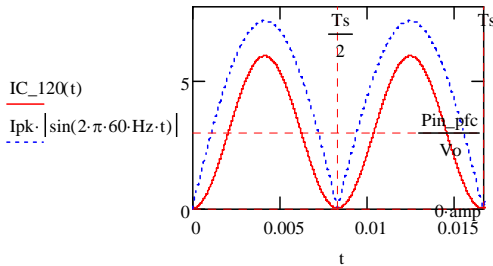


Figure 6-Low Frequency Output Capacitor Current (Co) Vs Low Frequency Inductor Current

We now take this current, $IC_{120}(t)$, and calculate the capacitor ripple current for using the following integral (note here we subtract out the DC component before integrating):

$$V_c(t) := \frac{1}{C_{actual} \cdot 2} \int_{-t}^t (IC_{120}(t) - IC_{PK_120Hz}) dt \quad (10)$$

We are now ready to plot the line frequency output bus ripple voltage with its DC component ($B(t) = V_o + V_c(t)$): The phase response for this signal relative to the input line current is as important as its gain response since it will become an operand in our multiplier block.

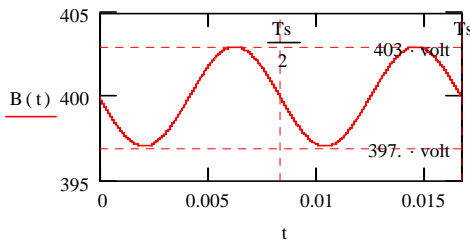


Figure 7-Output Bus Ripple Voltage

The above signal feeds into our output error amplifier via divider resistor chain (R_1 and R_2). To determine the output response we again find the Fourier coefficient matrix of $V_c(t)$, or equation (10), and also evaluate the transfer

function matrix of the feedback amplifier section for

$$s_k := j \cdot 2 \cdot \frac{\pi}{T_s} \cdot k$$

$K=1$ to 40 (note we only expect 2nd harmonic here).

$$c_k := \frac{1}{T_s} \int_{0 \text{-sec}}^{T_s} V_c(t) \cdot e^{-j \cdot k \cdot 2 \cdot \frac{\pi}{T_s} \cdot t} dt \quad (11)$$

Now we algebraically determine the transfer function of our error amplifier for $k=1$ to 40. The result is as follows:

$$Toac_k := \frac{R_f}{(1 + R_f \cdot s_k \cdot C_f) \cdot R_2} \quad (12)$$

And for the output response as before we take the following product and plug into the time domain Expansion and plot (where $U_0=400V$, $R_f=221K$, $R_2=790K$, $R_1=10K$, $C_f=2200pF$, $V_{ref}=5V$):

$$Voac_k := c_k \cdot Toac_k \quad (13)$$

Where our DC component is expressed with the following equation (which takes into account the working linear range of the L4981 IC): This DC component varies with load as expected.

$$U_0 := 1.8 \cdot \text{volt} \cdot \frac{Pin_{pfc}}{1250 \cdot \text{watt}} + 1.28 \cdot \text{volt} \quad (14)$$

Now we can write an equation and plot $VEA(t)$:

$$VEA(t) := U_0 + \sum_k \left(2 \cdot \text{Re}(Voac_k) \cdot \cos\left(2 \cdot k \cdot \frac{\pi}{T_s} \cdot t\right) \right) - \sum_k \left(2 \cdot \text{Im}(Voac_k) \cdot \sin\left(2 \cdot k \cdot \frac{\pi}{T_s} \cdot t\right) \right) \quad (15)$$

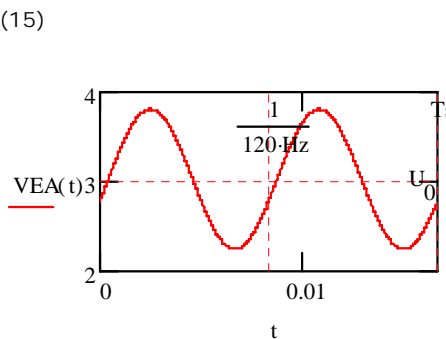


Figure 8- Error Amplifier 2nd harmonic ripple.

IV. L4981 MULTIPLIER BLOCK EQUATION AND WAVEFORMS:

Since we now have characterized $V_{ff}(t)$ and $V_{EA}(t)$, we can finally proceed with writing the equation³ which governs the internal multiplier of the L4981 PFC IC. According to the application notes for this device, and taking into account the value of our PFC shunt resistor (0.025 OHMS) along with $R_{prog}=7500$ OHM, we can write an expression for the predicted input line current:

$$I_{predict}(t) := \frac{V_{EA}(t) - 1.28 \cdot \text{volt}}{V_{FF}(t)^2} \cdot \frac{I_{ac}(t) \cdot R_{prog}}{.025 \cdot \text{ohm}} \cdot .1 \cdot \text{volt} \quad (16)$$

Where $I_{ac}(t)$ is the input wave-shaping signal as expressed by:

$$I_{ac}(t) := \left| \frac{V_{in} \sqrt{2} \cdot \sin\left(2\pi \frac{t}{T_s}\right)}{R_{ac}} \right| \quad (17)$$

For the values given above, the following is a plot of our input current as compared to the ideal case with zero distortion: Note that our selection of component values above cause significant distortion to the input current. Also note the higher peak value.

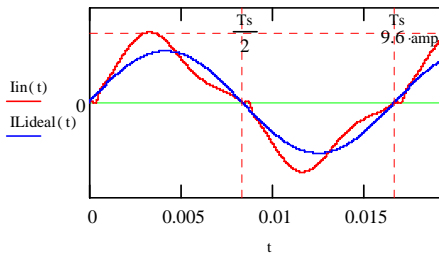


Figure 9- Predicted Input Line Current Vs Non-Distorted Case

V. Zero-Crossing Distortion Modeling

Another form of distortion associated with these types of regulators is zero-crossing or sometimes noted as “cusp” distortion. In actuality, this distortion becomes a problem at low line and high power levels. At the lower line, the current amplifier saturates near the zero-crossings since there is insufficient input voltage to support the required amount of current for the given inductance and output power levels. This distortion usually occurs right after respective zero crossings and can be represented by the function below. An expression valid only for a single-cycle can be written using the unit step response. If we take this new expression and multiply it with $I_{predict}(t)$, we will get a realistic amount of distortion near the zero-crossings

(below graph is for $d=5\%$); The %d term can be varied to either maximize or minimize its effect. Experimental results show that this value ranges inversely from 2% at 230 VRMS to 5% at 85V RMS input line with a typical value of 400uH for our boost inductor at an output power of roughly 1K Watts. The plot in figures 10 and 11 below are for $d=5\%$.

$\Phi(t)$ =unit step function

$$df(t) := \Phi\left(t - T_s \cdot d\right) - \Phi\left(t - \frac{1}{2} \cdot T_s\right) + \Phi\left[t - \frac{1}{2} \cdot T_s \cdot (1 + 2 \cdot d)\right] - \Phi(t - T_s) \quad (18)$$

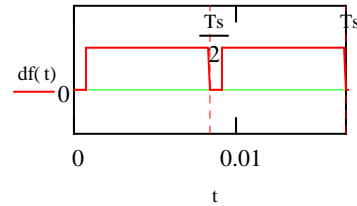


Figure 10- Plot of Zero-Crossing Multiplier

$$I(t) := df(t) \cdot I_{predict}(t) \quad (19)$$

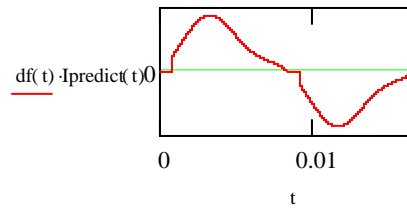


Figure 11- Plot of Zero Crossing Distortion

We can now take (18) above and multiply it with $I_{predict}(t)$ before taking the final Fourier transforms as detailed in the next section. Note that equation (19) is valid only over a single cycle. After performing a Fourier transform, the zero-crossings becomes periodic.

VI. INPUT CURRENT DISTORTION ANALYSIS AND IEC61000-3-2

Now for the last stage of our analysis we will determine the harmonic RMS levels of our input line current waveform or $I_{predict}(t)$ shown in figure 11 above (note this time we will use the trigonometric form of the Fourier Transform -either exponential or trigonometric form will yield the same result).

$$a_k := \frac{2}{T_s} \int_{0 \text{ sec}}^{T_s} I_{\text{predict}}(t) \cdot df(t) \cdot \cos\left[2 \cdot k \cdot \left(\frac{\pi}{T_s} \cdot t\right)\right] dt \quad (20)$$

$$b_k := \frac{2}{T_s} \int_{0 \text{ sec}}^{T_s} I_{\text{predict}}(t) \cdot df(t) \cdot \sin\left[2 \cdot k \cdot \left(\frac{\pi}{T_s} \cdot t\right)\right] dt \quad (21)$$

$$\text{dist}_k := \frac{1}{2} (a_k - j \cdot b_k) \quad (22)$$

Now we can plot the equation for the predicted input line current $I_{in}(t)$ using the time domain expansion and verify it against actual currents on the input line (this waveform is a valid periodic expression whereas the waveform in Figure 11 is single-cycle expression). Note the multiple cycles and the rounding of the edges of zero-crossings.

$$I_{in}(t) := \sum_k \left(2 \cdot \text{Re}(\text{dist}_k) \cdot \cos\left(2 \cdot k \cdot \frac{\pi}{T_s} \cdot t\right) \right) - \sum_k \left(2 \cdot \text{Im}(\text{dist}_k) \cdot \sin\left(2 \cdot k \cdot \frac{\pi}{T_s} \cdot t\right) \right) \quad (23)$$

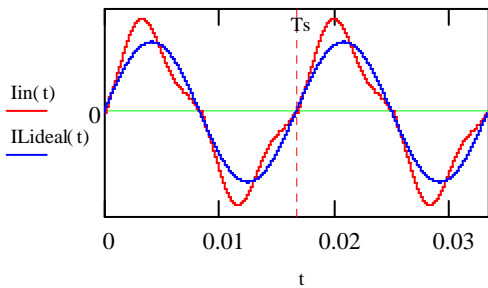


Figure 12- Fourier Version (40 harmonics) of I_{predict}

We now finally plot the harmonic levels of this input waveform (figure 11) using the following expression (24) along with the IEC61000-3-2 limits as shown in Table 1 below in the frequency domain: Since the IEC limits are measured in RMS we must apply the necessary conversion factor required to ensure that our harmonics are also in RMS.

$$\text{dist}_{\text{rms}_k} := \text{dist}_k \sqrt{2} \quad (24)$$

IEC61000-3-2 Class A – It is the “catch all” category. It includes motor driven equipment with phase angle control, most domestic appliances, and virtually all three phase equipment (<=16 Amps per phase). Anything that does not fit into the other three classes is also automatically categorized as Class A equipment. The limits are only defined for 230 V single phase and 230/400 V three-phase equipment⁴.

The Maximum current allowed for each harmonic, to meet the Class A limits, is shown in Table 1.

Table 1 Class A Equipment:

Harmonic Order (n)	Maximum permissible harmonic current (Amperes)
Odd Harmonics	
3	2.30
5	1.14
7	0.77
9	0.40
11	0.33
13	0.21
15 <= n <= 39	0.15 x (15/n)
Even Harmonics	
2	1.08
4	0.43
6	0.30
8 <= n <= 40	0.23 x (8/n)

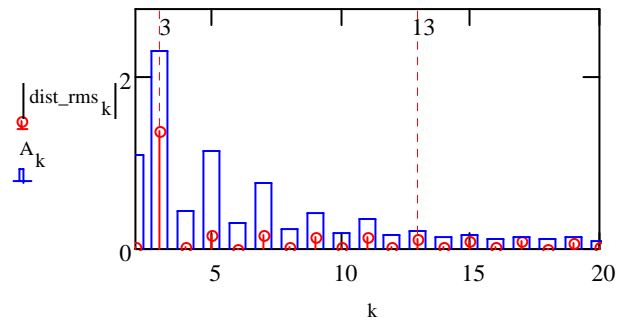


Figure 13-Frequency Plot of Input Line Current Vs. IEC61000-3-2 Limits (K=1 through 20):

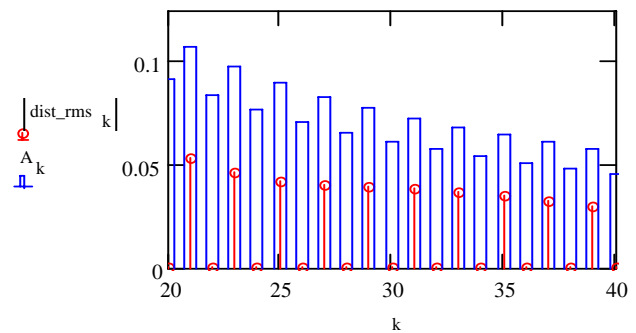


Figure 14-Frequency Plot of Input Line Current Vs. IEC61000-3-2 Limits (K=20 through 40):

The above frequency response is the Fourier transform of the waveform in shown figure 12. As can be seen in the graph, there is significant 3rd harmonic. This is mainly due

to the feedback loop component values since the feed-forward values for C1 and C2 are 1uF for this example. These are fairly large feed-forward caps and result in a good amount of filtering. In fact, careful consideration must be paid to these feed-forward values since there are higher order harmonics associated with the sharp edges of the rectified line as well as the fact that this signal eventually gets squared before being inverted and fed into the multiplier. In any case, the higher order harmonics (beyond 3rd) seen in the above spectrum is mainly associated with the zero-crossings. One way to check the integrity of the transform equations [20-24] is to highly filter the feedback and feed-forward values and verify that the fundamental component I_1 converges to the input RMS = $\frac{I_{L_60Hz_pk}}{\sqrt{2}}$

Converter Engine and Multiplier Components:

Pin=1205 W; Vin=230 VRMS; Vo=400V;d=3%;
Co=1344uF; Rac=900K OHM;Rprog=7.5K OHM;
Rshunt=.025 OHM;

Feed-Forward Component Values:

R1=282K; R2=15K;R3=7.5K;C1=1uF; and C2=0.1uF;

Feed-Forward Component Values:

R1=10K; R2=790K; Rf=221K;Cf=2200pF;

The final two calculations are power factor and THD and are as follows: We can also compare these results with measurements that are obtained with a power meter to verify convergence:

$$PF := \frac{Pin_pfc}{\sqrt{\frac{2}{T_s} \int_{0\text{-sec}}^{T_s} I_{in}(t)^2 dt \cdot Vin}} \quad (25)$$

$$THD := \frac{1}{|dist_1|} \cdot \sqrt{\sum_k (|dist_k|)^2 - (|dist_1|)^2} \quad (26)$$

THD = 22.36%

VII. EXPERIMENTAL RESULTS:

In order to compare our prediction against actual values, we will compare our results for 2 cases. **Case 1** is where the feedback loop dominates the input distortion and **Case 2** is where our feed-forward loop values predominate. We will intentionally use component values that will sufficiently distort the input line in order to see plainly whether we have good convergence.

CASE 1 (Feedback Dominant Case):

Converter Engine and Multiplier Components:

Pin=1205 W; Vin=230 VRMS; Vo=400V;d=3%;

Co=1344uF; Rac=900K OHM;Rprog=7.5K OHM;
Rshunt=.025 OHM;

Feed-Forward Component Values:

R1=282K; R2=15K;R3=7.5K;C1=1uF; and C2=0.1uF;

Feed-Forward Component Values:

R1=10K; R2=790K; Rf=221K;Cf=2200pF

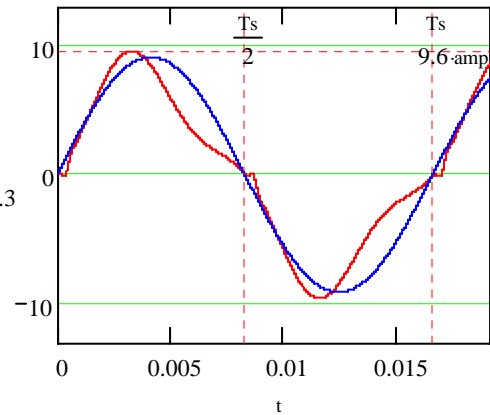
Power Factor:

Predicted PF-	P.F.=0.861
Measured PF-	P.F.=0.912
Percent Deviation-	Delta<6%

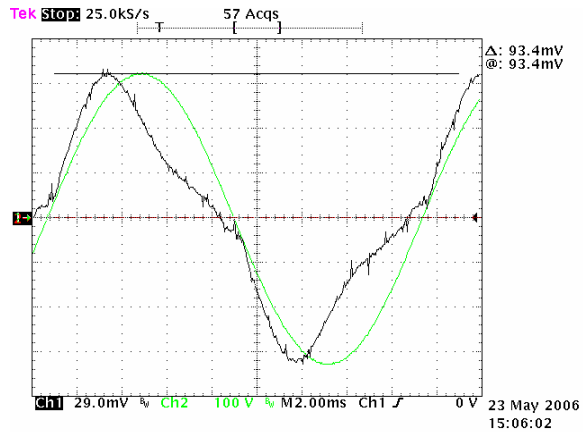
Total Harmonic Distortion:

Predicted THD-	THD=23.2%
Measured THD-	THD=22.36
Percent Deviation-	Delta<4%

Case 1 Predicted Input Current $I_{in}(t)$ vs. Normalized



Input Sine wave (this was scaled to match peaks).



Case 1 Actual Input Line Current $I_{in}(t)$ vs. Input Line Voltage. Ch1=10A/Volt; CH2=100V/Div;

Note: The peak line level closely correlates to the prediction (10A; CH1), though here is a very slight fundamental phase shift between the predicted vs. actual. This is due to the input line filter which our simulation does not include. In general we are getting excellent correlation for this condition.

CASE 2 (Feed-forward Dominant Case):

Converter Engine and Multiplier Components:

Pin=1205 W; Vin=230 VRMS; Vo=400V;d=3%;
Co=1344uF; Rac=900K OHM;Rprog=7.5K OHM;
Rshunt=.025 OHM;

Feed-Forward Component Values:

R1=282K; R2=15K;R3=7.5K;C1=1uF; and C2=0.1uF;

Feed-Forward Component Values:

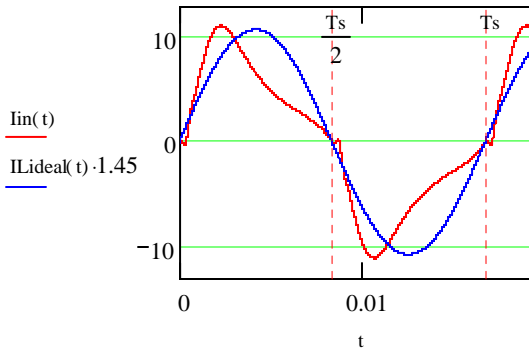
R1=10K; R2=790K; Rf=221K;Cf=2200Pf;

Power Factor:

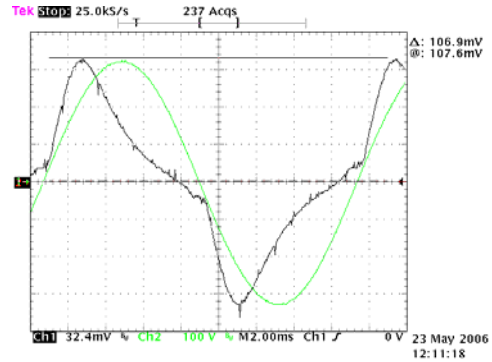
Predicted PF-	P.F.=0.826
Measured PF-	P.F.=0.844
Percent Deviation-	Delta=2.1%

Total Harmonic Distortion:

Predicted THD-	THD=32.26%
Measured THD-	THD=32.84%
Percent Deviation-	Delta<2%



Case 2 Predicted Input Current $I_{in}(t)$ vs. Normalized Input Sine wave (this was scaled to match peaks).



Case 2 Actual Input Line Current $I_{in}(t)$ vs. Input Line Voltage. Ch1=10A/Volt; CH2=100V/Div;

Note: Again the peak line level closely correlates to the prediction (11A; CH1). We still see the phase shift as expected from the input filter. Again we are getting excellent correlation for this condition as well.

VIII. CONCLUSION:

In this paper, a mathematical approach to analyzing the input line current harmonics for low distortion rectifiers was proposed. Time domain expressions and plots for the various signals present at the feedback and feed forward points, among other important locations, were presented. Power factor and THD were calculated as well. Finally, the input current was plotted via the use of the Fourier transform, in the frequency domain and compared against the IEC61000-3-2 limits up to and including the 40th harmonic of line frequency. Our experimental results of THD and power factor showed very tight correlation (at roughly 6% worst-case) between theoretical and empirical results. Excellent correlation was also noted between predicted and actual input current waveforms as well.

REFERENCE:

- [1] G. Spiazzi, P. Mattavelli, & L. Rossetto, Power factor preregulators with improved dynamic response, Proc. IEEE PESC, 1995, Atlanta, Georgia, P150–156.
- [2] R. Gabel, R. Roberts, Signals And Linear Systems 2nd Edition, P266-281.
- [3] AN827, A 500W High Power Factor With The L4981A Continuous Mode IC, 1995, 2003, P 10.
- [4] Supratim Basu¹, M.H.J.Bollen², Tore M.Undeland, PFC Strategies in light of EN 61000-3-2, APEC paper A123656.